Dudley's representation theorem in infinite dimensions and weak characterizations of stochastic integrability

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Joint work with Martin Ondreját

Introduction

- $(\Omega, \mathscr{A}, \mathbb{P})$ - a probability space
- $\mathcal{F} = (\mathscr{F}_t)_{t\geq 0}$ - filtration
- W - a (cylindrical) Brownian motion with respect to $\mathcal{F}$.

Let $I : L^p_{\mathscr{F}^W}(\Omega; L^2(\mathbb{R}_+)) \to L^p(\Omega, \mathscr{F}^W_\infty, \mathbb{P})$ be defined by

$$I\phi = \int_0^\infty \phi \, dW.$$

(i) I is an isomorphism onto $L^p_0(\Omega, \mathscr{F}^W_\infty, \mathbb{P})$ for $p \in (1, \infty)$,

(ii) I is a surjection when $0 \leq p < 1$,

(iii) I is an isomorphism onto the Hardy space H^1 when $p = 1$.

Results in (i) are connected to the martingale representation theorem.
In (ii) there is no uniqueness: there exists $\phi \in L^0_{\mathscr{F}^W}(\Omega; L^2(0,1))$:

$$\int_0^1 \phi \, dW = 0 \quad \text{and} \quad \int_0^{1/2} \phi dW = W(1/2)$$

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History and main problem

(i) is due to Itô '51, Kunita-Watanabe '67 and Garling '78.

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(iii) is due to Dubins, Gilat, Jacka, Oblój, Revuz, Yor, etc.

- van Neerven-Weis-V. [Ann. Prob. 07]: E-valued extension of (i) under geometric condition UMD on the Banach space E.

Main question in the first part of this talk:

- Does (ii) extend to the vector-valued setting?

In other words: is it true that for every $\xi \in L^0(\Omega, \mathscr{F}_\infty^W; E)$ there exists a $\phi \in L^0_{\mathscr{F}^W}(\Omega; L^2(\mathbb{R}_+; E))$ such that $\int_0^\infty \phi \, dW = \xi$?

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Overview

Dudley's result in infinite dimensions

Theorem (Ondreját-V. (J. Theor. Prob. '13))

Let E be a Banach space and $0 < b \leq \infty$. Let $\xi : \Omega \to E$ be strongly measurable. The following are equivalent:

1. *there exists a stochastically integrable $\phi : [0, b] \times \Omega \to E$ such that*

$$\int_0^b \phi \, dW = \xi$$

2. *ξ is $\sigma(\mathcal{F}_t : t < b)$-measurable.*

- $\mathcal{F}^W$-measurability is not needed in this result.
- Only $\mathcal{F}_\infty$-measurability of ξ required if $b = \infty$.
- If additionally $\xi \in L^p(\Omega; E)$ for $p \in (0, 1)$, then one can construct ϕ with additional p-integrability properties.

Dudley's result in infinite dimensions

One of the key lemmas (based on the ideas of Dudley '77):

Lemma

Let $0 \leq a < b \leq \infty$. Let $f_b(x) = (b - x)^{-1}$. Let $\eta : \Omega \to \mathbb{R}$ be $\mathcal{F}_a$-measurable. Let Y, h and τ be defined by

$$Y(r) = \int_a^r f_b(s)\, dW(s), \quad h(r) = \int_a^r f_b(s)^2\, ds$$

$$\tau = \inf\Big\{ r \geq a : Y(r) = \eta \Big\}$$

Then $a \leq \tau < b$ a.s. and

$$\sqrt{\frac{2}{\pi e}}\, \mathbb{E} \min\left\{ \frac{|\eta|}{\sqrt{t}}, 1 \right\} \leq \mathbb{P}(h(\tau) > t) \leq \mathbb{E} \min\left\{ \frac{|\eta|}{\sqrt{t}}, 1 \right\}, \quad t > 0.$$

Dudley's result in infinite dimensions

Idea of the proof of the representation theorem (no technicalities):

Choose $(a_n)_{n\geq 1}$ and $(\xi_n)_{n\geq 1}$ such that

$$(a_n)_n \text{ increasing}, \ \xi_n \text{ is } \mathcal{F}_{a_n} - \text{measurable and } \|\xi_n - \xi\|_0 \leq 4^{-n}.$$

Here $\|\eta\|_0 = \mathbb{E}(\|\eta\|_E \wedge 1)$. Now let

$$\phi_n(t) = (\xi_n - \xi_{n-1})\mathbf{1}_{(a_n,\tau_n)}(t)(a_{n+1} - t)^{-1}$$

where $\tau_n \in [a_n, a_{n+1})$ as in the previous lemma with $\eta = 1$. Then

$$\int_0^\infty \phi_n \, dW = \xi_n - \xi_{n-1}, \quad \Big\| \int_0^t \phi_n \, dW \Big\| = \Big| \int_0^t \|\phi_n\| \, dW \Big|.$$

Moreover, $\phi = \sum_{n\geq 1} \phi_n$ is stochastically integrable and,

$$\int_0^\infty \phi \, dW = \sum_{n=1}^\infty \xi_n - \xi_{n-1} = \xi \quad \text{in } L^0(\Omega; E).$$

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Universal representation theorem

Theorem (Ondreját-V. '13)

Let E be a separable Banach space. Assume $\mathcal{F}_\infty$ is $\mathbb{P}$-countably generated. Then there exists a strongly predictable process $\phi : \mathbb{R}_+ \times \Omega \to E$ which is locally stochastically integrable and

- *for every $\xi \in L^0(\Omega, \mathcal{F}_\infty; E)$ there exists an increasing sequence $(n_k)_{k\geq 0}$ of natural numbers such that*

$$\lim_{k\to\infty} \zeta(n_k) = \xi \quad \text{in } L^0(\Omega; E),$$

  where $\zeta(t) = \int_0^t \phi \, dW$.

- Result is new even in the scalar setting.
- ϕ is only locally integrable, because $\lim_{t\to\infty} \zeta(t)$ does not exist.
- There is also a version of the result on finite time intervals.

Proof based on similar ideas as previous result.

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Weak characterizations

Representation problems are closely connected with the problem of weak characterization of stochastic integrability.

- E be a Hilbert space, $p \in [0, \infty)$,
- ϕ a strongly $\mathcal{F}$-progressively measurable E-valued process
- $\xi \in L^p(\Omega; E)$

Assume $\langle \phi, x \rangle \in L^p(\Omega; L^2(0,1))$ for all $x \in E$ and

$$\int_0^1 \langle \phi(s), x \rangle \, dW(s) = \langle \xi, x \rangle, \quad x \in E. \tag{1}$$

Does this imply that ϕ is stochastically integrable in E and,

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Weak characterizations

Answer is known in several cases:

- Finite dimensional case.
- For $p > 1$ this result is due to van Neerven-Weis-V. '07
- In particular, the assumptions imply $\phi \in L^p(\Omega; L^2(0,1;E))$ and then also $\int_0^1 \phi(s)\,dW(s) = \xi$ (thus weak=strong).
- A version of the result holds for UMD spaces E.

We will discuss remaining cases:

- $p = 1 \to$ positive answer
- $0 \leq p < 1 \to$ negative answer

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Counterexample for $p < 1$

Theorem (Ondreját-V '13)

Let E be an infinite dimensional Hilbert space. There exists a strongly progressive process $\phi : [0,1] \times \Omega \to E$ with $\langle \phi, x\rangle \in L^p(\Omega; L^2(0,1))$ for all $x \in E$ and for all $p \in [0,1)$, and

$$\int_0^1 \langle \phi(t), x\rangle \, dW(t) = 0 \quad \text{for all } x \in E \tag{2}$$

but $\|\phi\|_{L^2(0,1;E)} = \infty$ almost surely.
In particular, ϕ is not stochastically integrable.

This provides a counterexample to the weak characterization of stochastic integrability. In this case $\xi = 0$.

- Construction based on techniques from Dudley's representation theorem and the existence of $\phi \neq 0$ with $\int_0^1 \phi \, dW = 0$.

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Counterexample for $p < 1$

One of the additional ingredients (suitable for probability exam):

Lemma

Let $(\xi_n)_{n\geq 1}$ be a sequence of independent $[0,\infty)$-valued random variables for which there is a constants $c, C > 0$ such that

$$c(t+1)^{-1} \leq \mathbb{P}(\xi_n > t) \leq C(t+1)^{-1}, \quad t \geq 0.$$

For a sequence $(c_n)_{n\geq 1}$ of positive numbers TFAE:

1. $\sum_{n\geq 1} c_n < \infty$.
2. *for all (for some) $p \in (1,\infty]$, $\|(c_n\xi_n)_{n\geq 1}\|_{\ell^p} < \infty$ a.s.*

$$\mathbb{E}[(c_n\xi_n)^p \wedge 1] = \int_0^1 \mathbb{P}((c_n\xi_n)^p > t)\, dt \leq C\int_0^1 \frac{c_n}{t^{1/p} + c_n}\, dt \leq C' c_n.$$

Summation over n yields (1) $\Rightarrow$ (2).

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Positive result for $p = 1$

Theorem (Ondreját-V. '13)

Let E be a Hilbert space. Let $\phi : [0, T] \times \Omega \to E$ be strongly progressively measurable. Assume $\langle \phi, x\rangle \in L^1(\Omega; L^2(0, T))$ for all $x \in E$. Let $\xi \in L^1(\Omega; E)$. If

$$\int_0^T \langle \phi, x\rangle \, dW = \langle \xi, x\rangle, \quad \text{for all } x \in E.$$

then ϕ in $L^0_{\mathcal{F}}(\Omega; L^2(0, T; E))$, and $\int_0^T \phi \, dW = \xi$. Moreover, for all $r \in (0, 1)$ one has $\phi \in L^r(\Omega; L^2(0, T; E))$.

- Version of the result holds for UMD spaces E.
- Proof based on Burkholder-Davis-Gundy for stochastic integrals.

Doob's representation theorem

Theorem (Ondreját-V. '13)

Let E be a UMD Banach space. Let $M : \mathbb{R}_+ \times \Omega \to E$ be a continuous local martingale. If there exists a progressively measurable $\phi : \mathbb{R}_+ \times \Omega \to E$ such that $\langle \phi, x^* \rangle \in L^0(\Omega; L^2(\mathbb{R}_+))$ for all $x^* \in E^*$, and for all $x^* \in E^*$, a.s.

$$[\langle M, x^* \rangle]_t = \int_0^t |\langle \phi(s), x^* \rangle|^2 \, ds, \quad t \in \mathbb{R}_+,$$

then there exists a Brownian motion W on an extended probability space such that ϕ is stochastically integrable and a.s.

$$M_t = \int_0^t \phi(s) \, dW(s), \quad t \in \mathbb{R}_+. \tag{3}$$

Summary and remarks

- Dudley's representation theorem holds for any Banach space E
- Weak characterizations hold for processes ϕ which are weakly in $L^p(\Omega; L^2(0,1))$ and random variables $\xi \in L^p(\Omega; E)$ as long as $p \in [1, \infty)$. For $p < 1$, there are counterexamples.
- Doob's representation theorem naturally extends to UMD spaces.
- What about weak characterizations for integrators different from Brownian motion ?

Thank you for your attention.

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